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Too little or too much R&D?

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Abstract

According to the first generation models of endogenous growth based on expanding product variety, the market economy unambiguously generates too little R&D. Later, by disentangling returns to specialization from the market power parameter, it was shown that with sufficiently low returns to specialization too much R&D can occur. The present paper takes a step further, disentangling the market power parameter from the capital share in final output. We show that this helps finding too much R&D as well. In addition, by differentiating between net and gross returns to specialization it is demonstrated what drives the differing inefficiency results in this literature. The decisive factor behind excessive R&D is the implicit presence of negative externalities of increased specialization. Empirically, an advantage of the more general framework is better agreement with the observed level of markups and the observed falling tendency of the patent/R&D ratio.

JEL Classification: O33, O41

Key words: Endogenous growth; Research and development; Expanding product variety; Surplus appropriability problem; Creative destruction; Optimal growth.

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1 Introduction

The purpose of this paper is to differentiate a few central parameters that were rigidly linked in the Romer model of endogenous growth. Through this differentiation the possibility arises that the market economy has too much research, as in the "quality ladder" type of models of endogenous growth. The central point can be summarized with the help of the aggregate production function

$$Y = A^{\eta} X^{\alpha} {N_Y}^{1-\alpha}, \qquad 0 < \alpha < 1, \quad \eta > 0,$$

where A is a measure of the level of technical knowledge in society or the stock of engineering principles that grows through research. Each "unit of knowledge" corresponds to a design of a specialized capital good. The parameter η captures the "returns to specialization", i.e., the degree to which society benefits from specializing production in an increasing number of branches. N_Y is the labor input, and X is a CES aggregate of quantities, x_i, of specialized capital goods:

$$X = A(\frac{1}{A}\sum_{i=1}^{A} {x_i}^{\varepsilon})^{\frac{1}{\varepsilon}}, \qquad 0 < \varepsilon < 1,$$

where ε can be called the "substitution parameter".

The original Romer (1990) article had implicitly the three parameters linked by $\eta = 1 - \alpha$ and $\varepsilon = \alpha$. As a result, the model had the particular feature that the amount of research is always insufficient. In later articles Benassy (1998) and Groot and Nahuis (1998) showed that if η and α are chosen independently, then too much research can occur if η is sufficiently low[1]. The present paper shows that differentiating between ε and α helps finding too much research as well. Further, by differentiating between net and gross returns to specialization we are able to demonstrate what it is that drives the differing inefficiency results in this literature. Empirically, an advantage of the more general framework is better agreement with the observed level of markups and the observed falling tendency of the patent/R&D ratio.

This paper also considers whether the parameter separations and allowance of a "lot of increasing returns" conflict with sufficient conditions for an optimal solution

[1] On the face of it, neither the Benassy or the Groot and Nahuis paper might seem to fit into this framework since they are based on the Grossman and Helpman (1991a) expanding consumer goods variety model without physical capital. As we shall see, however, the reduced form of their (essentially identical) models corresponds exactly to the case $\varepsilon = \alpha$.

to the social planner's problem and whether uniqueness of this solution and of the market solution always holds.

On one point our model has similarity with the Chapter-5 version of the increasing variety model in Grossman and Helpman (1991b). That model also implies a separation of the substitution parameter from the capital share in manufacturing. But the expanding product variety feature is limited to non-durable intermediate goods, and returns to specialization are implicitly given as a function of the share of these intermediate goods and the markup parameter. This leads to the reappearance of the Romer result that there is always too little R&D generated under laissez-faire.

Our paper is related to Jones and Williams (2000) who also, in a model of semi-endogenous growth, among other things loosen the usual parameter links. But this is done only halfway, and the three central entities – returns to specialization, monopolist markup, and capital share – are still linked in an arbitrary way. The focus is not on the analytical relationships opened up by parameter separations, but on calibrating the model for the US economy. The conclusion is that the decentralized economy vastly underinvests in R&D relative to what is socially optimal. Stokey (1995), which is a paper in the "quality ladder" tradition, is less firm about this matter.

The remainder of the present paper is organized as follows. Section 2 presents the elements of our extended Romer model. Section 3 considers the control problem of the social planner and characterizes its solution. Section 4 embeds the economic system into a market economy. In addition, this section shows how different contributions to the literature can be seen as special cases of the model. In Section 5 we compare the balanced growth properties of the market economy with those of the social optimum. Section 6 relates the model to the empirics of markups and the trend of the patent/R&D ratio. Finally, Section 7 concludes.

2 Elements of the economy

The economy is populated by a constant number, L, of infinitely lived identical households with constant size. Each household supplies one unit of labor inelastically. Their preferences can be represented by a discounted utility function,

$$U_0 = \int_0^\infty e^{-\rho t} \frac{c^{1-\theta} - 1}{1-\theta} dt, \quad \rho > 0, \theta > 0, \tag{2.1}$$

where c is consumption at time t, θ is the elasticity of marginal utility, and ρ is the rate of time preference [2].

The economy has two production sectors: the basic-goods (or "manufacturing") sector and the specialized capital-goods sector. In the *basic-goods sector,* labor and specialized capital goods are the inputs to produce the aggregate output, Y, according to the production function presented in the introduction. It is repeated here for convenience:

$$Y = A^\eta X^\alpha {N_Y}^{1-\alpha}, \qquad 0 < \alpha < 1, \quad \eta > 0. \tag{2.2}$$

The technical designs and the corresponding specialized capital goods are numbered chronologically, $i = 1, 2, ..., A$. The parameter η captures the degree to which society benefits from specializing production in an increasing number of branches[3]. Each new branch corresponds to a new design, that is a new specialized capital good. The composite factor X is given by

$$X = A(\frac{1}{A} \sum_{i=1}^{A} {x_i}^\varepsilon)^{\frac{1}{\varepsilon}}, \qquad 0 < \varepsilon < 1. \tag{2.3}$$

Thus the existing specialized capital goods exhibit a constant (direct) elasticity of substitution given by $\frac{1}{1-\varepsilon} > 1$ implying that no specialized capital good is essential. A higher ε indicates larger substitutability between the specialized capital goods; hence, we call ε the "substitution parameter". Notice that (2.3) inserted into (2.2) shows the specialized capital goods to be complements in the production of basic goods ($\frac{\partial^2 Y}{\partial x_i \partial x_j} > 0$) if $\alpha > \varepsilon$ and to be supplements ($\frac{\partial^2 Y}{\partial x_i \partial x_j} < 0$) if $\alpha < \varepsilon$.

The output of basic goods is used for consumption, $C \equiv cL$, and investment in "raw" capital. The stock of raw capital K changes according to

$$\dot{K} = Y - C - \delta K, \qquad \delta \geq 0, \; K(0) = K_0, \tag{2.4}$$

where a dot over a variable signifies the derivative with respect to time t, δ is the capital depreciation rate, and K_0 is a given positive number.

[2] In case $\theta = 1$, the expression $\frac{c^{1-\theta}-1}{1-\theta}$ should be interpreted as $\ln c$.

[3] More precisely, η is *net* returns to specialization, cf. Section 5.2.

In the *specialized capital-goods sector,* which is also the *innovative sector,* a unit of raw capital can immediately be transformed to a specialized capital good on the basis of a given technical design. The number of new designs created per time unit is assumed proportional to research work. And research efficiency is assumed proportional to the existing stock of knowledge, measured by A (this is the standing-on-shoulders effect, also present in Romer 1990, Benassy 1998, and Groot and Nahuis (1998). Hence, ignoring indivisibility,

$$\dot{A} = \gamma N_A A, \qquad \gamma > 0, \quad A(0) = A_0, \tag{2.5}$$

where A_0 is a given positive number, γ is a productivity parameter, and N_A is aggregate research work. Finally, with full employment,

$$N_Y + N_A = L. \tag{2.6}$$

Because of the strict concavity of X in x_i and the symmetric cost structure, static efficiency requires $x_i = x$ for $i = 1, 2, ...A$.[4] Hence, assuming static efficiency, $X = Ax$ from (2.3). Further, the stock demand for raw capital can be written Ax and when demand equals supply we have

$$X = Ax = K. \tag{2.7}$$

Inserting into (2.2) this gives output of basic goods as

$$Y = A^{\eta} K^{\alpha} {N_Y}^{1-\alpha}. \tag{2.8}$$

A feasible path $(K, A, C, Y, N_Y, N_A)_{t=0}^{\infty}$ is called a *steady state* if K, A, C, and Y are strictly positive and grow at constant though not necessarily equal (or positive) rates. Let the rate of growth of a strictly positive variable x be denoted g_x, i.e., $g_x \equiv \dot{x}/x$. Let u be the fraction of total labor supply employed in the basic-goods sector, i.e., $N_Y \equiv uL$, $0 \leq u \leq 1$. By (2.5), $g_A \geq 0$ always.

Lemma 1 *(i) In a steady state with* $g_A = \bar{g}_A$, $u = 1 - \frac{\bar{g}_A}{\gamma L}$, *a constant. Moreover,* $0 < u \leq 1$ *and* $0 \leq \bar{g}_A < \gamma L$. *(ii) If, in addition,* Y/K *is constant*[5], *then* $g_c = g_Y = g_K \equiv \bar{g} = \frac{\eta}{1-\alpha}\bar{g}_A$.

Proof. See Appendix 8.1. ■

[4] Therefore, obsolescence of old capital goods never occurs.

[5] A steady state commonly has constant Y/K. But there is an exception. If $I = 0$, i.e., $C = Y$ and $g_K = -\delta$, then, by (2.8), $g_C = g_Y = \eta g_A + \alpha g_K = \eta g_A - \alpha\delta > -\delta$ when $\delta > 0$; hence, Y/K increases.

3 The social optimum

The social planner will of course ensure static efficiency. Therefore, in the social optimum, output of basic goods is given by (2.8). The social planner chooses a path $(c, N_Y)_{t=0}^{\infty}$ to maximize U_0 subject to (2.4), (2.5), (2.6), (2.8), and the non-negativity requirements: $A, K \geq 0$ for all $t \geq 0$.

From the first order conditions for an interior solution we find (see appendix 8.1) the rate of growth of consumption to be[6]

$$g_c^* = \frac{1}{\theta}(\frac{\eta}{1-\alpha}\gamma L - \rho). \tag{3.1}$$

Hence, the optimum rate of growth does not depend on the substitution parameter ε; this parameter gets a role only through the market forces considered in the next section. However, an increase in the returns to specialization parameter, η, raises the degree to which society benefits from new inventions, which leads to an increase in g_c^*. In addition, the inverse of the labor share in manufacturing output acts as a "multiplier" transforming returns to specialization into an elasticity of labor efficiency with respect to technical knowledge. Given η, this elasticity decreases when the labor share in output, $1-\alpha$, increases, thereby lowering the growth rate in a steady state. Moreover, the higher is the desire for smoothing consumption (higher θ) and the higher is the rate of impatience (larger ρ), the smaller is g_c^*. In addition, the growth rate increases with the size of population; this is the well-known, though controversial[7], "scale effect" of R&D-based endogenous growth models.

In the derivation of the expression for g_c^* above, an interior steady state is presupposed[8]. That is, the inequalities $0 < g_c^* < \eta\gamma L/(1-\alpha)$ are presupposed (the second inequality arising from (ii) of Lemma 1 combined with (2.5)). Using (3.1), we see that $g_c^* < \eta\gamma L/(1-\alpha)$ holds if and only if

$$\rho > (1-\theta)\frac{\eta}{1-\alpha}\gamma L, \tag{A1}$$

and $g_c^* > 0$ holds if and only if

$$\rho < \frac{\eta}{1-\alpha}\gamma L. \tag{A2}$$

[6] Variables describing the social planner's solution are marked by *.

[7] Cf. Jones (1995).

[8] A steady state with positive employment in both sectors is called an interior steady state.

The conclusion is that, given A1 and A2, there exists a unique steady state[9], and it satisfies $0 < g_c^* < \frac{\eta}{1-\alpha}\gamma L$. The steady state is saddlepoint stable, implying existence and uniquenes of a converging path; further, this path *is* indeed an optimal solution (Appendix 8.1)[10].

If, however, A1 is violated, the rate of time preference is so small that the system cannot avoid the temptation to specialize in R&D activity forever (thus postponing production of consumption goods forever); in this case no optimal solution exists. On the other hand, if A2 is violated, then impatience is so large that there will be no R&D activity and no growth in a steady state. The formula (3.1) is no longer valid; instead, the steady state solution of the model will be like that of a standard one-sector Ramsey model without technical progress[11].

Now, we will embed the economic system in a market economy. Apart from the more general specifications of technology, the set-up is similar to Romer (1990).

4 The market economy

4.1 Agents

The representative firm in the basic goods sector rents labor at the wage w and specialized capital goods at the rental rate R_i, $i = 1, 2, ..., A$. Using basic goods as our *numeraire,* profit maximization under perfect competition yields

$$\frac{\partial Y}{\partial N_Y} = (1-\alpha)\frac{Y}{N_Y} = w, \tag{4.1}$$

$$\frac{\partial Y}{\partial x_i} = \alpha \frac{Y}{X}\frac{\partial X}{\partial x_i} = R_{i.} \tag{4.2}$$

From (4.2) we can express the demand for the specialized capital good i conditional on a given X as:

$$x_i = \frac{X}{A}(\frac{R_i}{R})^{-\frac{1}{1-\varepsilon}}, \qquad i = 1, 2, ..., A, \tag{4.3}$$

[9] We define uniqueness of a steady state to be present if, given the parameters $(\alpha, \varepsilon, \theta, \rho, \eta, \delta, \gamma, L)$, there exists only one pair $(g_c, Y/K)$ which is consistent with the steady state conditions.

[10] In view of the parameter separations made (allowing more "non-convexities" than usual) existence and uniqueness were not obvious beforehand.

[11] As is well-known, also this steady state is unique.

where $R = (\frac{1}{A}\sum_{i=1}^{A} R_i^{\frac{\varepsilon}{\varepsilon-1}})^{\frac{\varepsilon-1}{\varepsilon}}$ is the "ideal" price index for X (the minimum cost per unit of X).

The supply of each specialized capital good is decided by the firm that invented the design for the capital good in question, i.e., firm i supplies capital good variety i. The firms get compensated for the sunk research cost through retention of monopoly power over the commercial use of the invention and this monopoly power is supported by patents of infinite duration[12]. Given design i, to deliver $x(i)$ units of capital good i, it takes $x(i)$ units of raw capital. At each instant of time, firm i, facing the demand curve (4.3) and taking X and R as given, sets the rental rate R_i so that current profit $\pi_i \equiv R_i x_i - (r+\delta)x_i$ is maximized,

$$R_i = \frac{1}{\varepsilon}(r+\delta), \tag{4.4}$$

where r is the real rate of interest. Smaller ε (indicating less substitutability between specialized capital goods) gives larger monopoly power to the suppliers of specialized capital goods.

Since, by (4.4), all firms in the specialized capital-goods sector set the same rental price $R \equiv \frac{1}{\varepsilon}(r+\delta)$, they supply the same quantity, x, and they earn the same profit

$$\pi = \frac{1}{\varepsilon}(r+\delta)x - (r+\delta)x = (\frac{1}{\varepsilon} - 1)(r+\delta)x. \tag{4.5}$$

The equilibrium value of a patent is given by the present discounted value of the revenue that the patent generates

$$p(t) = \int_t^\infty (\frac{1}{\varepsilon} - 1)(r(\tau)+\delta)x(\tau)e^{-\int_t^\tau r(s)ds}d\tau. \tag{4.6}$$

Differentiating this expression with respect to t (using Leibniz' rule) yields the no-arbitrage condition

$$\frac{\pi + \dot{p}}{p} = r, \tag{4.7}$$

i.e., the return on a patent must be equal to the return on capital.

There is free entry to research activity. Research is done by new firms wanting to enter the specialized capital-goods sector. Given the invention production function

[12] At initial time, $t = 0$, the number of firms, $A(0)$, is large enough so that each firm's action is negligible in the aggregate economy.

(2.5), the value of the marginal product of labor in research is $p\gamma A$. Hence, profit maximization subject to (2.5) entails, in equilibrium,

$$w \geqq p\gamma A, \qquad \text{with} = \text{if } N_A > 0. \tag{4.8}$$

Once a new technical design has been invented, a patent is taken out and the new firm starts supplying the corresponding new specialized capital good. Although the right to supply a specific capital good is patented, everybody has access to the scientific principle behind the new design, i.e., technical knowledge is considered only partially excludable. In other words, by increasing A, research activity has a positive external effect on the productivity of future research activity[13]. In addition, research activity has a positive overall effect on total factor productivity in manufacturing (through the term A^{η} in (2.8)).

Households consume and save, and savings can be either in capital or in shares of the monopoly firms. Financial wealth of the representative household is $v \equiv \frac{K+pA}{L}$. The household makes a plan $(c)_{t=0}^{\infty}$ to maximize U_0 subject to

$$\dot{v} = w + rv - c, \quad v(0) = v_0, \tag{4.9}$$

$$\lim_{t\to\infty} ve^{-\int_0^t rds} \geq 0, \tag{4.10}$$

where v_0 is a given number, and the last constraint is the no-Ponzi-game condition. Necessary and sufficient conditions for a solution are that the Keynes-Ramsey rule,

$$g_c = \frac{1}{\theta}(r - \rho), \tag{4.11}$$

and the transversality condition,

$$\lim_{t\to\infty} ve^{-\int_0^t rds} = 0, \tag{4.12}$$

hold for all $t \geq 0$.

4.2 General equilibrium

Clearing conditions in the capital and labor markets entail: $K = X = xA$, $L = N_A + N_Y$. We define the *effective* capital-labor ratio in the basic-goods sector as

[13] Each research firm is small and therefore perceives, correctly, its contribution to aggregate $\dot{A}$ to be practically negligible.

$\tilde{k} \equiv K/(A^{\frac{\eta}{1-\alpha}}uL)$, where u is the fraction of total labor supply employed in the basic-goods sector. Then

$$x = \frac{K}{A} = uL\tilde{k}A^{\frac{\eta+\alpha-1}{1-\alpha}}. \tag{4.13}$$

Output per unit of *effective* labor in the basic-goods sector is $\tilde{y} \equiv Y/(A^{\frac{\eta}{1-\alpha}}uL)$ $= \tilde{k}^{\alpha}$. With these definitions and the clearing conditions together with (4.1), (4.2), and (4.4), we have

$$w = (1-\alpha)\frac{Y}{uL} = (1-\alpha)\tilde{k}^{\alpha}A^{\frac{\eta}{1-\alpha}}, \tag{4.14}$$

$$\frac{1}{\varepsilon}(r+\delta) = \frac{\partial Y}{\partial K} = \alpha\frac{Y}{K} = \alpha\tilde{k}^{\alpha-1}. \tag{4.15}$$

If $u < 1$, i.e., $N_A > 0$, then w also equals the value of the marginal product of labor in research so that (4.8) reduces to $w = p\gamma A$. This together with (4.14) gives the market value of a patent as

$$p = \frac{1-\alpha}{\gamma}\tilde{k}^{\alpha}A^{\frac{\eta+\alpha-1}{1-\alpha}}. \tag{4.16}$$

To summarize, a perfect foresight equilibrium, henceforth just an *equilibrium,* for this market economy is a path for prices and quantities such that for all $t \geq 0$: (i) consumers maximize discounted utility, i.e., they satisfy the Keynes-Ramsey rule and the transversality condition, taking the path of interest rates and wage rates as given; (ii) basic-goods producers maximize profits choosing inputs of labor and a list of specialized capital goods taking the prices of these inputs as given; (iii) each firm in the specialized capital-goods sector chooses a rental price to maximize profits, taking as given the interest rate and the downward-sloping demand curve it faces, (iv) R&D activity takes place only when (4.8) holds with equality; (v) in each market, the supply is equal to the demand. An *interior equilibrium* is an equilibrium such that all $t \geq 0$, $0 < u < 1$ (there is positive employment in both sectors).

4.3 Steady state

In Appendix 8.2 the growth rate of consumption in an interior steady state is shown to be

$$g_c = \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{1-\varepsilon\alpha-(1-\theta)\eta}\frac{\eta}{(1-\alpha)}. \tag{4.17}$$

This formula presupposes two parameter restrictions. First, the transversality condition (4.12) will be satisfied if and only if

$$\rho > \frac{1-\theta}{1-\varepsilon\alpha}\frac{\eta}{1-\alpha}(1-\varepsilon)\alpha\gamma L. \tag{A3}$$

If this does not hold, then the rate of time preference is so small that the market economy tends to grow at a rate above the interest rate, and household wealth tends to infinity, thus violating the equilibrium assumption.

An equally important parameter restriction comes from the interiority condition $u < 1$ (or $g_A > 0$) which holds if and only if

$$\rho < \frac{1}{1-\alpha}(1-\varepsilon)\alpha\gamma L. \tag{A4}$$

If, however, A4 is violated, then impatience is so large that R&D activity and growth cannot be supported in a steady state equilibrium. In this case the steady state solution of the model is like that of a one-sector model without technical progress. Also this zero-growth steady state is unique (since $\frac{Y}{K} = \frac{1}{\alpha\varepsilon}(r+\delta) = \frac{\rho+\delta}{\alpha\varepsilon}$ from (4.15) and (4.11)).

Observe that the parameter restriction A4, not containing either η or θ, is of a quite different nature compared to A2 of the previous section[14]. On the other hand, if A1 is assumed, we don't have to worry about A3. Indeed:

Lemma 2 *(i) A1 implies A3; (ii) A3 and A4 imply that* $1-\varepsilon\alpha > (1-\theta)\eta$*; (iii) A1 and A3 are satisfied automatically when* $\theta \geq 1$*.*

Proof. See Appendix 8.2 ■

The implication of (ii) of the lemma is that, taken together, A3 and A4 rule out the possibility of a non-positive denominator in (4.17).

The conclusion is that, given A3 and A4, there exists a unique steady state, and it satifies $0 < g_c < (1-\varepsilon)\alpha\gamma L/(1-\varepsilon\alpha)$. The steady state can be shown to be saddlepoint stable, at least within the empirically relevant domain of the parameter space (see Appendix 8.2).

From expression (4.17) follows that returns to specialization, η, the rate of impatience, ρ, the desire for consumption smoothing, θ, and the size of population, L, affect the growth rate qualitatively as in the social optimum (the derivations

[14] The implications of this are laid open in Section 5.

are in appendix 8.2). New features are, however, *first*, that the capital share parameter α affects growth through an additional channel compared with the social optimum. Indeed, the higher is α, the lower is the wage share in manufacturing, $1-\alpha$, which, given the factor prices, implies less room for profitable employment in the manufacturing sector[15]. Thereby more of the fixed labor force is available for employment in research, and growth is enhanced. The *second* new feature is that the growth rate of the market economy depends (negatively) on the substitution parameter ε which the social optimum did not. When specialized capital-goods are close substitutes (ε high), the markup over marginal cost in the specialized capital-goods sector becomes low, making inventions of new designs less profitable, thereby reducing growth. These two new features come from effects on private incentives.

4.4 Earlier contributions as special cases

The Romer (1990) model is simply the case $\eta = 1-\alpha$, and $\varepsilon = \alpha$[16]. The restriction $\eta = 1-\alpha$ has the implication that the value p of a patent and the size x of the market for a specific capital good stay constant in a steady state. To compare, if $\eta \gtrless 1-\alpha$, respectively, then p and x would increase/decrease, respectively, cf. (4.16) and (4.13). The case $\eta = 1-\alpha$ is also a benchmark case in the sense that it implies that along a steady state path every new invention leaves profits π of the single monopoly firm unchanged; if, however, $\eta \gtrless 1-\alpha$, respectively, then π increases/decreases for every new invention as shown by (4.5), (4.13), and (4.15).

One problem with the restriction $\varepsilon = \alpha$ (also present in Barro and Sala-i-Martin, 1995, Chapter 6) is that it blurs the positive effect on growth of an increase in the monopolist markup. The restriction implies that a high markup, $1/\varepsilon$, goes with a low capital share and this overturns the markup effect on growth. Indeed, when $\varepsilon = \alpha$, (4.17) reduces to $g_c = \frac{\alpha\gamma L-\rho}{1-\alpha^2-(1-\theta)\eta}\eta$ so that $\frac{\partial g_c}{\partial(1/\alpha)} < 0$, given (A2) and (A4).

The contributions by Benassy (1998) and Groot and Nahuis (1998) have much similarity between them. They are based on essentially the same model (henceforth called the BGN model), that is, an extension of the Grossman and Helpman (1991a)

[15] (4.4), (4.14), and (4.15) give $uL = \frac{1-\alpha}{\alpha} K \frac{R}{w}$.

[16] Strictly speaking, this refers to the textbook version (Aghion and Howitt, 1998, p. 35 ff.). In the original version of the Romer model, the basic-goods sector also employs a fixed amount of a second type of labor called human capital, but this is of secondary importance in our context.

It should be recognized that Romer (1990, p. S81) actually encouraged an investigation of cases where $\varepsilon \neq \alpha$.

expanding consumer goods variety model without capital accumulation. The only input in production is labor. Hence, on the face of it the BGN model might not seem to fit into our framework. Nevertheless, as far as the reduced form (4.17) is concerned, the BGN model corresponds to the case $\varepsilon = \alpha$ and $\theta = 1$ (only logarithmic utility is considered). Indeed, the technology specification of the BGN model leads to $g_c = [(1-\omega)\gamma L - \omega\rho]\,\nu$, where ω is a substitution parameter such that the monopolist markup becomes $\frac{1}{\omega}$, while ν is the elasticity of labor efficiency with respect to technical knowledge (corresponding to our $\eta/(1-\alpha)$). With $\varepsilon = \alpha$ and $\theta = 1$, (4.17) gives $g_c = (\frac{\alpha}{1+\alpha}\gamma L - \frac{1}{1+\alpha}\rho)\nu$, which is the BGN formula when we put $\omega = \frac{1}{1+\alpha}$. As to the social optimum the BGN model has $g_c^* = \nu\gamma L - \rho$ which is the same as our result in Section 3 when $\theta = 1$. Extended to some general $\theta > 0$, the BGN formula becomes $g_c = \frac{(1-\omega)\gamma L - \omega\rho}{1-\omega(1-\theta)\nu}\nu$ for the market economy (from (4.17) with $\varepsilon = \alpha$) and $g_c^* = \frac{1}{\theta}(\nu\gamma L - \rho)$ for the social optimum.

The just mentioned Grossman and Helpman (1991a) model is a special case of the BGN model, namely the case $\nu = (1-\omega)/\omega$. As noted earlier, the book by Grossman and Helpman contains also an increasing variety model *with* capital accumulation (Grossman and Helpman, 1991b). This model can be seen as yet another special case of (the reduced form of) the BGN model. The formula $g_c = [(1-\omega)\gamma L - \omega\rho]\,\nu$ is still valid, but ω depends not only on the markup, but also on the share of capital in final output and the share of non-durable intermediate goods in final output[17]. Also, the knowledge elasticity parameter ν is a function of the markup and these two share parameters. The implied parameter links lead to the reappearance of the Romer result that there is always too little R&D generated under laissez-faire.

In the next section we study the necessary and sufficient conditions for the market economy to do too much R&D. We shall see how allowing for $\varepsilon \neq \alpha$ modifies these conditions compared with the BGN results. Since the BGN model can be seen as containing the two mentioned Grossman and Helpman contributions as special cases, our comparison with BGN will cover these contributions as well.

[17]In the Grossman and Helpman (1991b) model physical capital is a homogenous good while the specialized intermediate goods, sold under conditions of monopolistic competition, are non-durable.

5 Comparing market outcome and social optimum

5.1 Growth rates compared

The analysis of Sections 3 and 4 can be summarized in the following way:

Proposition 1 *(i) If A1 holds, there exits a unique steady state in the social optimum and it has*

$$g_c^* = \begin{cases} \frac{1}{\theta}(\frac{\eta}{1-\alpha}\gamma L - \rho) > 0, \quad \frac{\partial g_c^*}{\partial \eta} > 0, & \text{if } \rho < \frac{\eta}{1-\alpha}\gamma L, \\ 0 & \text{if } \rho \geq \frac{\eta}{1-\alpha}\gamma L, \end{cases}$$

(ii) If A3 holds, there exists a unique steady state in the market economy, and it has

$$g_c = \begin{cases} \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{1-\varepsilon\alpha-(1-\theta)\eta}\frac{\eta}{1-\alpha} > 0, \quad \frac{\partial g_c}{\partial \eta} > 0, \frac{\partial g_c}{\partial \alpha} > 0, \frac{\partial g_c}{\partial \varepsilon} < 0, & \text{if } \rho < \frac{1-\varepsilon}{1-\alpha}\alpha\gamma L, \\ 0 & \text{if } \rho \geq \frac{1-\varepsilon}{1-\alpha}\alpha\gamma L, \end{cases}$$

Proof. See text of Sections 3 and 4. ■

We now claim that there exists a critical value, $\tilde{\eta}$, such that if and only if the returns to specialization parameter is equal to this value, then $g_c = g_c^*$.

Proposition 2 *Assume A4. Then:*

(i) Given $(\alpha, \varepsilon, \theta, \rho, \gamma, L)$ there is a unique value $\tilde{\eta} > 0$ such that if $\eta = \tilde{\eta}$, then both the social optimum and the market economy have a steady state, and $g_c = g_c^*$.

(ii) The value $\tilde{\eta}$ satisfies the inequality $\frac{\rho}{\gamma L}(1-\alpha) < \tilde{\eta} < (1-\varepsilon)\alpha$.

(iii) For any η satisfying A1, both the social optimum and the market economy have a steady state, and the following rule applies: $g_c \gtreqless g_c^*$ for $\eta \lesseqgtr \tilde{\eta}$, respectively.

Proof. See Appendix 8.3. ■

In view of (4.5) and (4.15), profits per capital good can be written

$$\pi = (1-\varepsilon)\alpha\frac{Y}{A} = \frac{(1-\varepsilon)\alpha}{\eta}\partial Y/\partial A. \tag{5.1}$$

Therefore, a natural reference point for the returns to specialization parameter, η, is the number $(1-\varepsilon)\alpha$, henceforth called the "profit factor". To put the Romer and the BGN cases in perspective we observe:

Proposition 3 *Assume A4. Then:*

(i) The critical value $\tilde{\eta}$ *is below* $1-\alpha$ *in the Romer-BGN case* $\varepsilon = \alpha$.

(ii) A simple example where $\tilde{\eta}$ *is above* $1-\alpha$ *is:* $\theta = 1$, $\alpha > \max(\frac{\gamma L}{(1-\varepsilon)\rho+\gamma L}, \frac{\rho}{(1-\varepsilon)\gamma L+\rho})$.

(iii) For any $\eta > 0$ *such that the social optimum has a steady state, the market economy has a steady state with* $g_c > g_c^*$ *if and only if the profit factor* $(1-\varepsilon)\alpha > \max(\eta + \frac{\eta-\beta(1-\alpha)}{\beta(1-\alpha)-(1-\theta)\eta}(1-\alpha), \frac{\rho}{\gamma L}(1-\alpha))$.

Proof. See Appendix 8.3. ■

In the specific Romer case, in addition to $\varepsilon = \alpha$, $\eta = 1-\alpha$; hence, by (i) of Proposition 3, $\tilde{\eta} < 1-\alpha = \eta$, and we get the Romer conclusion that $g_c < g_c^*$ unambiguously. When $\varepsilon = \alpha$, the opposite inequality, $g_c > g_c^*$, arises, if and only if the knowledge elasticity η is considerably below $1-\alpha$; this confirms Benassy's and Groot and Nahuis's conclusion that excessive research in the market economy arises if and only if η is sufficiently below the Romerian value $1-\alpha$. However, as implied by (ii) and (iii) of Proposition 3, allowing for a separation of the substitution parameter, ε, from the share of capital, α, we have: For a sufficiently high capital share or a sufficiently high profit factor $(1-\varepsilon)\alpha$, g_c can end up larger than g_c^* even when η equals, or is above, $1-\alpha$. This confirms our claim in the introduction that by differentiating between ε and α, more scope for excessive research in the market economy is opened up.

The analyses of Benassy and Groot and Nahuis do not provide much intuition behind the excessive growth possibility. We shall now see that differentiating between net and gross returns to specialization will make clear *what* it is that drives the results. It turns out that there is much less asymmetry than hitherto recognized between the increasing variety models and the quality ladder models of endogenous growth.

5.2 Direct and indirect effects of specialization

We introduce a conceptual distinction between A, the number of existing different varieties at time t, and A_0, the number of these varieties being in use at time t (that is, the number of varieties for which $x_i > 0$). Consider the situation just before the invention of a new variety. Static efficiency requires $A_0 = A$ and $x_i = \bar{K}/A_0 \equiv x$, where $\bar{K}$ is a given aggregate amount of raw capital. We may write (2.2) as

$$Y = A^{\eta+(1-\frac{1}{\varepsilon})\alpha}(\sum_{i=1}^{A_0} x_i^{\varepsilon})^{\frac{\alpha}{\varepsilon}} N_Y^{1-\alpha} = A^{\eta+(1-\frac{1}{\varepsilon})\alpha} A_0^{\frac{\alpha}{\varepsilon}} x^{\alpha} N_Y^{1-\alpha} \equiv F(A, A_0, x, N_Y). \tag{5.2}$$

Now, *after* the invention of the new variety, static efficiency requires a *redistribution* of the given amount of raw capital to an enlarged spectrum of varieties. We call the effect on aggregate output of this redistribution the *direct effect* of increased specialization. This direct effect is

$$F_{A_0} + F_x \frac{\partial x}{\partial A_0} = (\frac{\alpha}{\varepsilon} - \alpha)\frac{Y}{A}. \tag{5.3}$$

In view of the assumption $0 < \varepsilon < 1$, this effect is always positive.

In general there is also an *indirect effect* of increased specialization, that is, an effect on the productivity of the already existing specialized capital goods. To calculate this effect, we freeze A_0 and x at the level they have before redistribution. Then we define the indirect effect as

$$F_A = \left[\eta + (1 - \frac{1}{\varepsilon})\alpha\right] \frac{Y}{A} \equiv \xi \frac{Y}{A}. \tag{5.4}$$

One can imagine this effect to be positive, zero, or negative depending on the circumstances. We shall speak of "creative synergy" when it is positive, and of "creative down-weighing" (a mild form of the "creative destruction" inherent in quality ladder models) when it is negative. The interpretation of the first case is that the direct contribution of the invention is complemented by the positive indirect effect due to the other capital goods becoming more productive when "assisted" by a more complete network of intermediate goods. The interpretation of the second case is that though the overall effect of an invention on capital productivity is positive (since $\eta > 0$, by assumption), the direct contribution of the invention is partly offset by a negative indirect effect due to, say, coordination difficulties in a more specialized and complex world. The indirect effect, whether positive or negative, appears in the market economy as an externality[18].

Expressing the direct and indirect productivity effects as elasticities, the overall effect of inventions can be written

$$\frac{A}{Y}\frac{\partial Y}{\partial A} = \text{direct effect} + \text{indirect effect} = (\frac{1}{\varepsilon} - 1)\alpha + \xi = \eta.$$

[18] Note that "creative down-weighing" refers to a reduction in the *productivity* of old capital goods. This need not imply reduced profits. Indeed, along a steady state path, profits per firm are reduced by the arrival of new inventions if and only if $\eta < 1 - \alpha$, cf. (4.5) and (4.13).

Thus, we may interprete returns to specialization η as a *derived* parameter, given α, ε, and the indirect effect ξ. When the indirect effect, ξ, is positive, we may speak of η as *total* returns to specialization. When ξ is negative, η may be called *net* returns to specialization while the direct effect, $(\frac{1}{\varepsilon} - 1)\alpha$, measures *gross* returns to specialization[19].

Romer's parameter links, $\eta = 1 - \alpha$ and $\varepsilon = \alpha$, imply a benchmark case where the indirect effect vanishes. But disentangling these parameter links leads to:

Proposition 4 *Assume A1 and A4 and let $\tilde{\eta}$ be defined as in Proposition 3. The case $g_c > g_c^*$ arises if and only if the indirect effect of inventions, ξ, on the productivity of old capital goods is negative and numerically above $(\frac{1}{\varepsilon} - 1)\alpha - \tilde{\eta}$.*

Proof. From (5.4) we have $\xi = \eta - (\frac{1}{\varepsilon} - 1)\alpha$. By (iii) of Proposition 3, $g_c > g_c^*$ if and only if $\eta < \tilde{\eta}$. Hence, $g_c > g_c^*$ if and only if $\xi < \tilde{\eta} - (\frac{1}{\varepsilon} - 1)\alpha$, where, since $0 < \varepsilon < 1$, $\tilde{\eta} - (\frac{1}{\varepsilon} - 1)\alpha < \tilde{\eta} - \varepsilon(\frac{1}{\varepsilon} - 1)\alpha = \tilde{\eta} - (1 - \varepsilon)\alpha < 0$ by (ii) of the Proposition 3. ■

To understand this result, notice that there are three potential market failures that may cause private and social incentives to diverge in the model. (i) The *intertemporal spillover*: By adding to the stock of technical knowledge, research increases the productivity of future research, cf. (2.5), but this effect is not compensated in the market. (ii) The *surplus appropriability problem*: Innovator's monopoly profits capture only a fraction of their (direct) contribution to output (compare (5.1) and (5.3))[20]. These two market failures are wellknown from the increasing variety literature as well as the quality ladder literature (Grossman and Helpman, 1991, Chapter 4) and Aghion and Howitt (1992, 1998). What is less wellknown is that the famous, additional market failure appearing in the quality ladder models, that of "creative destruction", may come up (in a related form) also in an increasing variety framework, that is as a negative ξ in 5.4. This *externality of innovations on the productivity of old capital goods* is here called market failure (iii). The market

[19] Consistency with our assumption that $\eta > 0$ requires $\xi > -(\frac{1}{\varepsilon} - 1)\alpha$.

[20] The surplus appropriability problem reflects that, from (4.15), capital costs are $r + \delta = \varepsilon \partial Y/\partial K < \partial Y/\partial K$. This inequality is a result of *monopoly pricing*: The markup implies a wedge between the price of the services of specialized capital goods and the marginal cost of providing them so that the demand for capital services is reduced. This also entails a wedge between social returns to saving, $\frac{\partial Y}{\partial K} - \delta$, and the private return, r, and therefore a tendency to too little saving. Hence, even in cases where $g_c = g_c^*$, the market economy underinvests in capital, implying too low consumption because of too low K/Y.

failures (i) and (ii) tend to generate insufficient research under laissez-faire, while market failure (iii) works in the same or the opposite direction, depending on the sign of ξ.

Now, combine this with Proposition 4 saying that unless there is *enough* "creative down-weighing", the market leads to underinvestment in R&D. To avoid this underinvestment, it is not enough that creative down-weighing, ξ, is numerically equal to or above $(1/\varepsilon - 1)\eta$ so that, by (5.4), the profit factor $(1-\varepsilon)\alpha \geq \eta$, hence, by (5.1), $\pi \geq \partial Y/\partial A$ (i.e., net surplus *is* appropriated). Indeed, this only ensures that the surplus appropriability problem is overcome, not that the intertemporal spillover is overcome.

The implied interpretation of the Romer-BGN parameter link $\varepsilon = \alpha$ is that this is a case where the necessary amount of creative down-weighing requires η to be considerably below the Romerian value $1 - \alpha$.

6 Remarks on the empirics

In addition to the richer set of theoretical possibilities, the more general framework used in this paper allows better accordance with the empirical evidence on markups and the trend of the patent-R&D ratio than does the more rigid Romer framework. Consider first the patent-R&D ratio, that is, the number of new patents per year divided by R&D expenditures. Since the late fifties, in the US a systematic decline in this ratio has taken place (on average a fall at 3.5 % per year, see Griliches, 1989, and Kortum, 1993). In our model, the patent-R&D ratio is given by $\dot{A}/(wN_A) = \gamma A/w$, and in a steady state this ratio will be decreasing over time if and only if (net) returns to specialization, η, is larger than $1-\alpha$ (because, in a steady state, by (4.14), $g_w = \frac{\eta}{1-\alpha} g_A$)[21]. But Romer-style models have $\eta = 1 - \alpha$, and they are therefore inconsistent with the observed fall in the patent-R&D ratio.

The markup is in Romer-style models implicitly given by the inverse of the capital share. The empirical evidence suggests that the capital share in output,

[21] To put it differently, the patent-R&D ratio falls when productivity in manufacturing increases faster than in R&D activity. This need not be a sign of exhaustion of technological opportunities. Rather than being something to worry about, it may, according to the present model, be a sign of high potentiality of new technical knowledge. (Of course, in reality the level of patenting lacks a lot in precision as an indicator of aggregate R&D successes inasmuch as many firms, at least outside the chemical and pharmaceutical industries, rely on other ways of protecting their innovations.)

α, is around 0.4 in the US (see, e.g., Cooley and Prescott (1995)) so that Romer-style models predict markups around 2.5! According to Norrbin (1993) and Basu (1996), however, markup estimations are between 1.05 and 1.40 in US industry. The present framework allows accordance with this, since we can choose a value for the substitution parameter ε in the interval $[0.70, 0.95]$.

These observations indicate the following size relation between the critical ("equal-growth") returns to specialization parameter, $\tilde{\eta}$, the direct effect of innovations, $(\frac{1}{\varepsilon}-1)\alpha$, and the actual value, η, of the returns to specialization parameter in US industry: $\eta > 1-\alpha > (\frac{1}{\varepsilon}-1)\alpha > \tilde{\eta}$ where the last two inequalities follow from $\varepsilon > \alpha$ and (ii) of Proposition 2, respectively. In particular, $\eta > (\frac{1}{\varepsilon}-1)\alpha$ is an indication that there is "creative synergy", and $\eta > \tilde{\eta}$ indicates that there is too little R&D so that the growth rate is inefficiently low. This result is consistent with the empirical evidence presented by Jones and Williams (1998). These authors find that the optimal R&D investment is at least four times larger than the actual spending[22]. Therefore, at least in this respect the prediction from the simple Romer framework with $\eta = 1-\alpha$ seems to point in the right direction. Our hint that $\eta > (\frac{1}{\varepsilon}-1)\alpha$ is likely, strengthens the confidence in that prediction[23].

7 Conclusions

According to the first generation of models of endogenous growth based on expanding product variety, the market economy unambiguously yields a too low level of R&D. However, disentangling returns to specialization from the market power parameter, later studies found that this result arises due to the implicit choice of a relatively high value for the returns to specialization.

The present paper takes a step further, analyzing an extended Romer-style model where returns to specialization, the monopolistic markup, and the capital share in final output are given by independent parameters. Previous expanding variety models can be seen as special cases. Among the implications of the extended model are

[22] Stokey (1995), a paper in the "quality ladder" tradition, is less firm about this vast underinvestment.

[23] It can be shown that an active government needs two instruments to establish the optimal allocation. These instruments could be, first, a subsidy to buyers of capital services in order to eliminate distorting demand effects of monopoly pricing, and, second, a tax on - or subsidy to - monopoly profits, depending on the parameters (if the above parameter values are accepted, a subsidy is indeed called for).

that low returns to specialization are not necessary for the case with too much R&D in the market economy to arise. Indeed, whatever the returns to specialization, the decisive factor is the implicit presence of enough negative externalities of increased specialization. These externalities are a reminiscence of the creative destruction effect in the quality ladder models. In this way, there seems to be much less asymmetry than hitherto recognized between the expanding variety models and the quality ladder models.

The parameter separations made allow more "non-convexities" than the usual models, but this is shown not to conflict with sufficient conditions for an optimal solution to the social planner's problem or with uniqueness of this solution. In addition, the market equilibrium is shown to be unique, at least within an empirically relevant domain of the parameter space.

A rudimentary calibration of the model suggests that the actual outcome for the US economy is not that of too much R&D, but that of too little R&D. Nevertheless, an advantage of the more general framework is better agreement with the observed level of markups and the observed falling tendency of the patent/R&D ratio.

8 Appendix

8.1 Technology. The social optimum

Proof of Lemma 1. (i) Consider a steady state with $g_A = \bar{g}_A$. Then, from (2.5), N_Y is constant. By definition of a steady state, $Y > 0$; hence, from (2.8), $N_Y > 0$. Therefore $u \equiv N_Y/L$ is constant, $u \in \left(0, 1 - \frac{\bar{g}_A}{\gamma L}\right]$, and $0 \leq \bar{g}_A = \gamma(1-u)L < \gamma L$. (ii) Y/K constant implies $g_Y = g_K$ which is constant in a steady state. Then, by (2.4), cL/K is constant, hence $g_c = g_K$. From (2.8), the common growth rate, $\bar{g}$, of c, K, and Y is $\frac{\eta}{1-\alpha}\bar{g}_A$. ■

Solution of the social planner's problem. In order to obtain concavity of the maximized Hamiltonian, we introduce the transformations $\tilde{A} \equiv A^{\frac{\eta}{1-\alpha}}$ and $\tilde{\gamma} \equiv \frac{\eta}{1-\alpha}\gamma$. Then $Y = K^\alpha(\tilde{A}N_Y)^{1-\alpha}$, and $\dot{\tilde{A}} = \tilde{\gamma}(L - N_Y)\tilde{A}$. The current value Hamiltonian for the social planning problem of Section 3 becomes

$$H = \frac{c^{1-\theta} - 1}{1-\theta} + \lambda_1[K^\alpha(\tilde{A}N_Y)^{1-\alpha} - \delta K - cL] + \lambda_2\tilde{\gamma}(L - N_Y)\tilde{A},$$

where λ_1 and λ_2 are the shadow prices of the state variables K and $\tilde{A}$, respectively. Necessary conditions for an interior solution are that for all $t \geq 0$:

$$c^{-\theta} = \lambda_1 L, \tag{8.1}$$

$$\lambda_1(1-\alpha)\frac{Y}{\tilde{A}N_Y} = \lambda_2\tilde{\gamma}, \tag{8.2}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \lambda_1(\frac{\partial Y}{\partial K} - \delta), \tag{8.3}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \lambda_1\frac{\partial Y}{\partial \tilde{A}} - \lambda_2\tilde{\gamma}(L - N_Y), \tag{8.4}$$

$$\lim_{t\to\infty} \lambda_1 e^{-\rho t}K = 0, \lim_{t\to\infty} \lambda_2 e^{-\rho t}\tilde{A} = 0. \tag{8.5}$$

Log-differentiating (8.1) wrt. t and using (8.3) gives

$$g_c \equiv \frac{\dot{c}}{c} = \frac{1}{\theta}(\frac{\partial Y}{\partial K} - \delta - \rho) = \frac{1}{\theta}(\alpha\tilde{k}^{\alpha-1} - \delta - \rho), \tag{8.6}$$

where $\tilde{k} \equiv k/\tilde{A}$. Since in a steady state, by definition, g_c is constant, $\tilde{k}$ is also constant in view of (8.6).

Define $\tilde{q} \equiv \lambda_2/\lambda_1$. From (8.2)

$$\tilde{q} = \frac{(1-\alpha)Y}{\tilde{\gamma}\tilde{A}N_Y} = \frac{1-\alpha}{\tilde{\gamma}}\tilde{k}^{\alpha}. \tag{8.7}$$

Hence, in steady state

$$\frac{\dot{\tilde{q}}}{\tilde{q}} = 0. \tag{8.8}$$

But, by definition of $\tilde{q}$, $\dot{\tilde{q}}/\tilde{q} = (\dot{\lambda}_2/\lambda_2) - (\dot{\lambda}_1/\lambda_1)$, hence, from (8.3), and (8.4),

$$\frac{\dot{\tilde{q}}}{\tilde{q}} = \frac{\partial Y}{\partial K} - \delta - \tilde{\gamma}L. \tag{8.9}$$

(8.8), (8.9), and the definition of $\tilde{\gamma}$ imply $\frac{\partial Y}{\partial K} = \frac{\eta}{1-\alpha}\gamma L + \delta$. Inserting this into (8.6) gives (3.1) in the text.

To show that the steady state is saddlepoint stable, let $z \equiv Y/K$ and $\bar{c} \equiv cL/K$. Then, from the first order conditions (8.1), (8.2), (8.3), (8.4), and the dynamic constraints of the social planner's problem we get the differential equations

$$\begin{aligned}
\dot{z} &= \left[(\alpha-1)z + \frac{1-\alpha}{\alpha}(\tilde{\gamma}L+\delta)\right]z, \\
\dot{\bar{c}} &= \left[\frac{\alpha-\theta}{\theta}z + \bar{c} + \frac{\theta-1}{\theta}\delta - \frac{\rho}{\theta}\right]\bar{c}, \\
\dot{u} &= \left[-\hat{c} + \tilde{\gamma}Lu + \frac{1-\alpha}{\alpha}(\tilde{\gamma}L+\delta)\right]u.
\end{aligned}$$

The Jacobian evaluated in the steady state is triangular and has the eigenvalues $\rho_1 = (\partial\dot{z}/\partial z)^* = (\alpha - 1)z^* < 0$, $\rho_2 = (\partial\dot{\bar{c}}/\partial\bar{c})^* = \bar{c}^* > 0$, and $\rho_3 = (\partial\dot{u}/\partial u)^* = \tilde{\gamma}Lu^* > 0$. The three variables, $z, \bar{c}$, and u, are jump variables, but the initial conditions for z and u are not independent. Given the predetermined variable $\bar{k} \equiv K/(\tilde{A}L)$, we have $u = \bar{k}z^{\frac{1}{1-\alpha}}$, from the production function $Y = K^\alpha(\tilde{A}N_Y)^{1-\alpha}$. Hence, with two free initial conditions and exactly two positive eigenvalues, existence and uniqueness of a convergent solution (that is, saddlepoint stability) holds generically.

A path $(c, N_Y, K, \tilde{A})_{t=0}^\infty$, which satisfies the first order and transversality conditions (8.1), (8.2), (8.3), (8.4), and (8.5), and which approaches the steady state, is our candidate for an optimal solution. To ensure that such a path is really an optimal solution, we check, first, whether the Hamiltonian is jointly concave in the state variables K and $\tilde{A}$ after the controls c and N_Y have been substituted by their maximizing values from (8.1) and (8.2), respectively. We find $H = B + \lambda_1\tilde{\gamma}LK + \lambda_2\tilde{\gamma}L\tilde{A}$, where the term B does not depend on K or $\tilde{A}$. Clearly this function is concave.

The second aspect to check is whether our candidate path $(c, N_Y, K, \tilde{A})_{t=0}^\infty$ satisfies a *sufficient* transversality condition. In view of (8.1) and (8.2), $\lambda_1 > 0$ and $\lambda_2 > 0$ for all $t \geq 0$. Hence, by (8.5), the path $(c, N_Y, K, \tilde{A})_{t=0}^\infty$ satisfies

$$\lim_{t\to\infty}\left[\lambda_1 e^{-\rho t}(\hat{K} - K) + \lambda_2 e^{-\rho t}(\widehat{\tilde{A}} - \tilde{A})\right] \geq 0,$$

for all feasible paths $(\hat{c}, \hat{N}_Y, \hat{K}, \widehat{\tilde{A}})_{t=0}^\infty$. The conclusion is that, by Arrow's sufficiency theorem (see Seierstad and Sydsæter, 1987, p. 236), our candidate path $(c, N_Y, K, \tilde{A})_{t=0}^\infty$ *is* an optimal solution.

8.2 The market economy

Steady state. In an interior equilibrium, by (4.16), the market value of a patent grows according to

$$g_p = \alpha g_{\tilde{k}} + \frac{\eta + \alpha - 1}{1 - \alpha} g_A. \tag{8.10}$$

Inserting this together with (4.14), (4.16), and (5.1) into (4.7), using the fact that $\gamma uL = \gamma L - g_A$, from (2.5), gives the market interest rate as

$$r = \frac{1 - \varepsilon}{1 - \alpha}\alpha(\gamma L - g_A) + (\frac{\eta}{1 - \alpha} - 1)g_A + \alpha g_{\tilde{k}}. \tag{8.11}$$

In a steady state, by definition c grows at a constant rate, hence, from (4.11), r is constant. Therefore, Y/K and $\tilde{k}$ are constant in view of (4.15), implying $g_c = \frac{\eta}{1-\alpha} g_A$ from Lemma 1. By (4.11) this implies

$$g_A = \frac{1-\alpha}{\eta\theta}(r-\rho). \tag{8.12}$$

An interior steady state has $u < 1$, hence (8.11) holds. Inserting the steady state condition $g_{\tilde{k}} = 0$ into (8.11) gives a "growth possibility curve" and combining this with the "growth preference curve", (8.12), leads to

$$g_A = \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{1-\varepsilon\alpha-(1-\theta)\eta}. \tag{8.13}$$

Multiplying by $\frac{\eta}{1-\alpha}$ gives the formula (4.17).

Interiority of the steady state implies, by definition, $0 < u < 1$ where

$$u = \frac{\left[1-\alpha-(1-\theta)\eta\right]\gamma L + (1-\alpha)\rho}{\left[1-\varepsilon\alpha-(1-\theta)\eta\right]\gamma L}, \tag{8.14}$$

from (8.13) and (2.5) with $N_A = (1-u)L$. Consider the "normal case" defined by the denominator of g_A being positive, that is

$$1-\varepsilon\alpha > (1-\theta)\eta. \tag{8.15}$$

Given (8.15), $u < 1$ holds if and only if

$$\rho < \frac{1-\varepsilon}{1-\alpha}\alpha\gamma L, \tag{A4}$$

and $u > 0$ if and only if

$$\rho > \left[(1-\theta)\frac{\eta}{1-\alpha} - 1\right]\gamma L. \tag{8.16}$$

However, for the steady state to be an equilibrium, the transversality condition (4.12) should be satisfied, and this requires u to have finite distance from 0. Indeed, $v \equiv \frac{K+pA}{L}$, and in steady state $g_K = g_c = \frac{\eta}{1-\alpha} g_A = g_p + g_A$, from (8.10); hence $g_v = g_c$. It follows that the steady state satisfies (4.12) if and only if the steady state has

$$r > g_c. \tag{8.17}$$

By (4.11), $r = \theta g_c + \rho$, and if $\theta \geq 1$, (8.17) holds automatically. Suppose, on the contrary, $\theta < 1$. The value of η for which $\theta g_c + \rho = g_c$ is, using $g_c = \nu g_A$, where g_A is given in (8.13),

$$\bar{\eta} \equiv \frac{(1-\varepsilon\alpha)(1-\alpha)\rho}{(1-\theta)(1-\varepsilon)\alpha\gamma L}. \tag{8.18}$$

Now, $0 < \frac{\partial r}{\partial \eta} = \theta \frac{\partial g_c}{\partial \eta} < \frac{\partial g_c}{\partial \eta}$ in this case, since, as shown below, $\frac{\partial g_c}{\partial \eta} > 0$. Therefore, (8.17) holds if and only if $\eta < \bar{\eta}$, and we have to strengthen the requirement $u > 0$ by substituting

$$\rho > \frac{1-\theta}{1-\varepsilon\alpha}\frac{\eta}{1-\alpha}(1-\varepsilon)\alpha\gamma L \tag{A3}$$

for (8.16). By (8.14), A4 together with A3 is equivalent to $\frac{1-\alpha}{1-\varepsilon\alpha} < u < 1$.

It follows that A3 guarantees $r > g_c$. Therefore, A3 gurantees the transversality condition of the household.

Proof of Lemma 2 (i) $\frac{(1-\varepsilon)\alpha}{1-\varepsilon\alpha} = \frac{\alpha-\varepsilon\alpha}{1-\varepsilon\alpha} < 1$; hence, A1 $\Rightarrow$ A3. (ii) A3 and A4 $\Rightarrow \frac{(1-\varepsilon)\alpha(1-\theta)}{(1-\alpha)(1-\varepsilon\alpha)}\eta < \frac{\rho}{\gamma L} < \frac{1-\varepsilon}{1-\alpha}\alpha$; hence, $(1-\theta)\eta < 1-\varepsilon\alpha$. (iii) This is obvious since $\rho > 0$. ■

Lemma A Assume A3 and (8.15). Then (8.16) holds.

Proof. (8.15) $\Rightarrow (1-\theta)\frac{\eta}{1-\alpha}(1-\varepsilon\alpha+\varepsilon\alpha-\alpha) < 1-\varepsilon\alpha \Rightarrow (1-\theta)\frac{\eta}{1-\alpha}\left[1-\frac{(1-\varepsilon)\alpha}{1-\varepsilon\alpha}\right] < 1 \Rightarrow (1-\theta)\frac{\eta}{1-\alpha} - 1 < \frac{(1-\varepsilon)\alpha}{1-\varepsilon\alpha}(1-\theta)\frac{\eta}{1-\alpha} < \frac{\rho}{\gamma L}$, by A3. ■

Varying the parameters. Assume A3 and A4. Then, by Lemma 2 and Lemma A, (8.15) and (8.16) hold. From (4.17) we get, after some manipulations,

$$\frac{\partial g_c}{\partial \eta} = \frac{(1-\varepsilon\alpha)\left[(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho\right]}{\left[1-\varepsilon\alpha-(1-\theta)\eta\right]^2(1-\alpha)} > 0,$$

by A4;

$$\frac{\partial g_c}{\partial \varepsilon} = \frac{\left[\gamma L((1-\theta)\eta-(1-\alpha)) - (1-\alpha)\rho\right]}{\left[1-\varepsilon\alpha-(1-\theta)\eta\right]^2(1-\alpha)}\alpha\eta < 0,$$

by (8.16); and

$$\frac{\partial g_c}{\partial \alpha} = \frac{\frac{1-\varepsilon}{1-\alpha}\gamma L\left[1-\varepsilon\alpha-(1-\theta)\eta\right] + \varepsilon\left[(1-\varepsilon)\alpha\gamma L-(1-\alpha)\rho\right]}{\left[1-\varepsilon\alpha-(1-\theta)\eta\right]^2(1-\alpha)}\eta > 0$$

by (8.15) and A4 applied to the first and the second square bracket, respectively, in the numerator.

Dynamics. The equations describing the dynamics of the market economy can be reduced to three differential equations in $z \equiv Y/K$, $\bar{c} \equiv cL/K$, and u. The determinant of the Jacobian evaluated at the steady state can be shown to be negative. Hence, either there are three eigenvalues with negative real part or one negative eigenvalue and two with non-negative real part. A sufficient though not necessary condition for the last case to obtain, is that $\eta \leq (1-\varepsilon\alpha)/(1-\alpha)$ *or* $\theta = 1$ (see Alvarez-Pelaez and Groth 2002). With $\alpha = .4$ and $\varepsilon < .95$, we have $(1-\varepsilon\alpha)/(1-\alpha) > 1.02$, a number that seems beyond realistic values of η. In view of the boundary condition, $u = \left[K/(A^{\frac{\eta}{1-\alpha}}L)\right] z^{\frac{1}{1-\alpha}}$, generically, this implies uniqueness of the convergent solution (that is, saddlepoint stability) at least within the empirically relevant domain of the parameter space.

8.3 Comparison

Proof of Proposition 2. Assume A4 and let $\beta \equiv \frac{\rho}{\gamma L}$.

(i) If the value $\tilde{\eta}$ exists, it must, by definition, satisfy A1; if it did not, we know from Section 3 that the social optimum would have no steady state. Now, consider an η such that A1 holds. In view of Lemma 2, A1 and A4 imply A3 so that for this η both the social optimum and the market economy have a unique steady state, by Proposition 1, and

$$\frac{g_c^*}{g_c} = \frac{[1-\varepsilon\alpha-(1-\theta)(1-\alpha)\nu]\,(\nu-\beta)}{[(1-\varepsilon)\alpha-(1-\alpha)\beta]\,\theta\nu} \equiv \varsigma(\nu;\theta), \tag{8.19}$$

where $\nu \equiv \frac{\eta}{1-\alpha}$. The condition $\varsigma(\nu;\theta) = 1$ implies the quadratic equation

$$Q(v) \equiv (1-\theta)\nu^2 - [1+\beta+(1-\theta)\phi(\varepsilon,\alpha)]\,\nu + \beta(1+\phi(\varepsilon,\alpha)) = 0,$$

where $\phi(\varepsilon,\alpha) \equiv \frac{1-\varepsilon}{1-\alpha}\alpha > 0$. For $\theta \neq 1$ the roots are

$$\begin{pmatrix}\nu_2\\ \nu_1\end{pmatrix} = \frac{1+\beta+(1-\theta)\phi(\varepsilon,\alpha) \pm \sqrt{D}}{2(1-\theta)}, \tag{8.20}$$

where

$$\begin{aligned} D &\equiv [1+\beta+(1-\theta)\phi(\varepsilon,\alpha)]^2 - 4(1-\theta)\beta(1+\phi(\varepsilon,\alpha)) \\ &= [1-\beta+(1-\theta)\phi(\varepsilon,\alpha)]^2 + 4\theta\beta > 4\theta\beta > 0. \end{aligned}$$

Let $\eta_i \equiv \nu_i(1-\alpha), i = 1, 2$.

Case 1: $\theta < 1$. From (8.20) follows $0 < \nu_1 < \nu_2$; we have $Q(\nu) < 0$ for $\nu_1 < \nu < \nu_2$ and $Q(\nu) > 0$ for $\nu < \nu_1$ and $\nu > \nu_2$. Since $Q(\frac{\beta}{1-\theta}) = \frac{-\theta\beta}{1-\theta} < 0$, $\nu_1 < \frac{\beta}{1-\theta} < \nu_2$. Therefore, η_1 satisfies A1, but η_2 does not satisfy A1 and can be discarded. Hence, $\tilde{\eta}$ is unique and equal to $\eta_1 \in (0, \frac{\beta(1-\alpha)}{1-\theta})$.

Case 2: $\theta > 1$. Now $D > [1 + \beta + (1-\theta)\phi(\varepsilon, \alpha)]^2$ so that

$$1 + \beta + (1-\theta)\phi(\varepsilon, \alpha) \pm \sqrt{D} \gtrless 0,$$

respectively. Hence, $\nu_2 < 0 < \nu_1$, and again η_2, can be discarded. We have $Q(\nu) > 0$ for $\nu_1 < \nu < \nu_2$ and $Q(\nu) < 0$ for $\nu < \nu_2$ and $\nu > \nu_1$. Again, $\tilde{\eta}$ is unique and equal to $\eta_1 > 0$.

Case 3: $\theta = 1$. In this case $Q(\nu) = 0$ has one root

$$\nu_1 (= \nu_2) = \frac{\beta}{1+\beta}(1 + \phi(\varepsilon, \alpha)), \tag{8.21}$$

and $Q(\nu) \gtrless 0$ for $\nu \lesseqgtr \nu_1$, respectively.

In all three cases, $\tilde{\eta}$ is unique and equal to $\eta_1 \equiv \nu_1(1-\alpha)$.

(ii) That $\tilde{\eta} < \frac{\beta(1-\alpha)}{1-\theta}$ when $\theta < 1$ (i.e., A1 holds), was shown above. If on the other hand $\theta \geq 1$, then, by choosing α sufficiently close to 1, A1 and A4 are still satisfied, and g_A comes arbitrarily close to $\frac{1}{\theta}\gamma L$, the limiting value of g_A^* for $\nu \to \infty$. That $\tilde{\eta} > \beta(1-\alpha)$ can be seen in the following way. By A4, $g_c > 0$; hence $\varsigma(\frac{\tilde{\eta}}{1-\alpha}; \theta) = 1 \Rightarrow g_c^* = g_c > 0$, i.e., $\tilde{\eta} > \beta(1-\alpha)$ from Proposition 1. To show that $\tilde{\eta} < \phi(\varepsilon, \alpha)(1-\alpha)$, consider

$$\begin{aligned} Q(\phi(\varepsilon, \alpha)) &= (1-\theta)(\phi(\varepsilon, \alpha))^2 - (1+\beta)\phi(\varepsilon, \alpha) - (1-\theta)(\phi(\varepsilon, \alpha))^2 + \beta(1 + \phi(\varepsilon, \alpha)) \\ &= \beta - \phi(\varepsilon, \alpha) < 0, \quad \text{by A4.} \end{aligned}$$

Hence, whether case 1, case 2, or case 3 above is true, we have $\nu_1 < \phi(\varepsilon, \alpha)$, hence, $\tilde{\eta} = \eta_1 = \nu_1(1-\alpha) < \phi(\varepsilon, \alpha)(1-\alpha) = (1-\varepsilon)\alpha$.

(iii) Consider a η such that A1 holds. In view of Lemma 2, A1 and A4 imply A3 so that for this η both the social optimum and the market economy have a unique steady state, by Proposition 1. Since, also from that proposition, g_c and g_c^* are increasing in η, we have $\eta \lesseqqgtr \tilde{\eta} \Rightarrow g_c \gtreqqless g_c^*$, respectively. ■

Proof of Proposition 3. Assume A4. (i) If $\varepsilon = \alpha$, then by (ii) of Proposition 2 we have $\tilde{\eta} < \alpha(1-\alpha) < 1-\alpha$. (ii) Let $\theta = 1$. Then, from (8.21) and the definition

$\phi(\varepsilon,\alpha) \equiv \frac{1-\varepsilon}{1-\alpha}\alpha$, $\tilde{\eta} = \nu_1(1-\alpha) = \frac{\rho}{\rho+\gamma L}(1-\varepsilon\alpha) \gtreqless 1-\alpha$ for $\alpha \gtreqless \frac{\gamma L}{(1-\varepsilon)\rho+\gamma L}$, respectively. But A4 holds if and only if $\alpha > \frac{\rho}{(1-\varepsilon)\gamma L+\rho}$. (iii) Consider a $\eta > 0$ such that the social optimum has a steady state. Then, from Section 3 we know that if $\theta < 1$, then $\eta < \frac{\rho(1-\alpha)}{(1-\theta)\gamma L}$, i.e., A1 is satisfied. In view of Lemma 2, A1 and A4 imply A3 so that for this η both the social optimum and the market economy have a unique steady state, by Proposition 1. Now, suppose $\eta > \frac{\rho(1-\alpha)}{\gamma L}$. Then, $\eta + \frac{\eta\gamma L-\rho(1-\alpha)}{\rho(1-\alpha)-(1-\theta)\eta\gamma L}(1-\alpha) > \eta > \frac{\rho(1-\alpha)}{\gamma L}$, and straightforward calculation from Proposition 1 gives

$$g_c > g_c^* \Leftrightarrow (1-\varepsilon)\alpha > \eta + \frac{\eta\gamma L - \rho(1-\alpha)}{\rho(1-\alpha)-(1-\theta)\eta\gamma L}(1-\alpha).$$

If instead, $\eta \leq \frac{\rho(1-\alpha)}{\gamma L}$, then $\eta + \frac{\eta\gamma L-\rho(1-\alpha)}{\rho(1-\alpha)-(1-\theta)\eta\gamma L}(1-\alpha) \leq \eta \leq \frac{\rho(1-\alpha)}{\gamma L}$. By Proposition 1, $g_c^* = 0$; but $g_c > 0$ if and only if $(1-\varepsilon)\alpha > \frac{\rho(1-\alpha)}{\gamma L}$, proving (iii). ■

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